Abstract Book of the International Workshop on Applied Geometry and Related Topics

14-18 July 2025

14 July (Monday)

Wave Pattern Solutions and Riemann Problem Analysis for the Zakharov-Ito System Using Whitham Modulation Theory

Yaqing Liu

Beijing Information Science and Technology University

This talk centers on the traveling wave solutions of the Zakharov-Ito (ZI) equation and the evolution of wave structures under initial discontinuity conditions. As an important integrable generalization of the KdV equation in (1+1)-dimensional space, the ZI equation holds considerable research value. Firstly, through Hamiltonian system analysis and phase portrait examination, a comprehensive classification of non-trivial traveling wave solutions for the ZI equation has been established, revealing the parametric dependence and dynamical characteristics of these solutions. Secondly, based on Whitham modulation theory and employing the finite-gap integration technique, the Whitham equations have been derived. It elucidates the essential characteristics of wave structure evolution, including the modulation of phase waves and the classification of solutions.

Stochastic multisymplectic PDEs and their structure-preserving numerical methods

Ruiao Hu

Imperial College London

We construct stochastic multisymplectic systems by considering a stochastic extension to the variational formulation of multisymplectic partial differential equations proposed in [Hydon, Proc. R. Soc. A, 461, 1627–1637, 2005]. The stochastic variational principle implies the existence of stochastic 1-form and 2-form conservation laws, as well as conservation laws arising from continuous variational symmetries via a stochastic Noether's theorem. These results are the stochastic analogues of those found in deterministic variational principles. Furthermore, we develop stochastic structure-preserving collocation methods for this class of stochastic multisymplectic systems. These integrators possess a discrete analogue of the stochastic 2-form conservation law and, in the case of linear systems, also guarantee discrete momentum conservation. The effectiveness of the proposed methods is demonstrated through their application to stochastic nonlinear Schrödinger equations featuring either stochastic transport or stochastic dispersion.

Recent Progress on the Hamel's Formalism for Classical Field Theory and Its Applications

Donghua Shi

Beijing Institute of Technology

To meet the demand for fast and qualitatively accurate algorithms in the dynamics computation and control of swarms and soft robots, the Hamel's formalism is derived within the framework of classical field theory using the moving frame method and exterior calculus. This formalism consists of the Hamel field equations characterized the dynamics, and geometric compatibility conditions, facilitating symmetry reduction and avoiding the need to solve high-complexity mixed-type equations.

Building upon this formulation, computational approaches employing discrete exterior calculus are developed to construct Hamel field integrators on fiber bundles. The efficacy of the proposed algorithms is shown through numerical simulations of geometrically exact beams/shells, and the electro-magneto-elastic coupled systems. Regarding control, a unified framework for flexible swarm formations is established, integrating reinforcement learning to develop flexible encirclement algorithms. Furthermore, an infinite dimensional controlled Lagrangian method is proposed, with applications demonstrated in the stabilization of flexible inverted pendulums.

Differentiable Geometry and Topology for Scalable 3D Shape Learning

Yihao Luo

Imperial College London

We present a series of methods that introduce differentiability into traditionally discrete geometric operations, enabling end-to-end optimization over 3D shapes. We begin with DOPH, a differentiable voxelization algorithm that supports backpropagation through mesh-to-volume mappings. This technique has broad applicability in deep learning for medical imaging and general 3D vision tasks. We further develop differentiable curvature estimators that robustly capture local geometric variations and can be seamlessly integrated into topology-aware learning constraints. Additionally, we introduce our recent works, MeshAnything V2&V3, which propose topology-aware mesh tokenization schemes tailored for LLM-based 3D generation. Together, these contributions advance scalable, structure-preserving, and semantically rich shape modeling across diverse 3D domains.

Molecular Simulation and ML-Based Phase Classification of Premelting Behavior at Ice Interfaces

Takumi Sato

Keio University

We investigated premelting behavior at ice–polymer interfaces using molecular dynamics simulations with the TIP4P/Ice model. Equilibrium structures of PS, PVA, and PEO were constructed, and

temperature-dependent density profiles were analyzed. At 259 K, hydrophilic polymers showed greater melting promotion than hydrophobic ones. To classify solid- and liquid-like regions in the premelting layer, an unsupervised deep learning model was developed using atomic displacement trajectories sampled from bulk solid and liquid simulations. The model outputs a scalar value to indicate phase state, enabling robust classification based on displacement statistics. This combined approach revealed how polymer hydrophilicity affects interfacial melting dynamics.

15 July (Tuesday)

Neural Fiber Geometry and Its Role in Information Processing, Higher Cognitive Functions, and Potential Links to Consciousness

Sehun Chun
Yonsei University

Contemporary AI technologies have largely evolved by digitizing neural networks with a primary focus on line-by-line connectivity. However, the spatial organization of the brain—particularly the complex geometry of neural fiber bundles—has received comparatively little attention. As a result, remarkable brain functions are often reduced in AI algorithms to oversimplified abstractions of the biological brain. A key missing component in understanding neural computation is the role of **ephaptic coupling**, where neural spike propagation is influenced by the spatial arrangement of fibers. Unlike cardiac signal propagation, neural signals travel through slender, one-dimensional fibers embedded in a three-dimensional extracellular matrix. This structural complexity, combined with the insulating myelin sheath and intermittent Nodes of Ranvier, gives rise to unique propagation dynamics within fiber bundles.

In this talk, I will introduce the fundamental principles of ephaptic coupling, present novel mathematical models and computational simulations, and discuss future directions—particularly those that may offer insights into the neural basis of human consciousness.

Deep Architecture from OT-Based Distribution Alignment for Cross-Modal Image Processing

Shihui Ying
Shanghai University

In this talk, we focus on the application of optimal transport theory and methods in the field of image processing. Starting from two tasks of fast cross-modal medical image reconstruction and natural scene semantic segmentation, we introduce optimal transport into the cross-modal generative process. Firstly, to address the challenge of fast cross-modal medical image reconstruction, we integrate spatial alignment and modality generation based on optimal transport representations. This approach establishes an interpretable deep learning framework for fast cross-modal MRI reconstruction. Secondly, for natural scene semantic segmentation, we overcome the high computational cost associated with traditional optimal transport in image space by introducing category-level optimal transport in the output space. This enables effective label propagation from the source domain to the target domain in natural scenes. Finally, experimental results on multiple datasets validate the effectiveness of the proposed methods.

The emergent of the relativistic Lagrangian from non-relativistic multiplicative Lagrangian: statistical and geometric perspective

Sikarin Yoo-Kong

Naresuan University

In this talk, we show that the relativistic Lagrangian for a free particle naturally emerges from the statistical average of the non-relativistic multiplicative Lagrangian. We shall point out that this construction reveals that the relativistic Lagrangian can be understood as emergent structures - not by imposing relativity externally, but by integrating over a parameter space of generalized classical models. This offers a novel bottom-up perspective: relativistic dynamics can emerge from hidden structure embedded in non-relativistic mechanic (Commonly, there is a topdown perspective through the limit c>>v.). Moreover, on the level of the action, statistical average gives rise to the relativistic action equipped with Minkowski's spacetime geometry in the case of 1+1 dimension. This, once again, reflects the bottom-up perspective, where spacetime astonishingly pops up as an emergent phenomenon.

Time series forecasting via a multiscale time-series-to-image encoding framework

Mingming Li

Yibin University

This talk proposes a novel framework that facilitates time series forecasting by integrating time-series-to-image encoding with adaptive image enhancement techniques. We introduce a multiscale Gramian Angular Field (GAF) method to transform raw temperature data into two-dimensional images at multiple temporal resolutions, preserving both short-term fluctuations and long-term trends. To address the challenge of mixed noise in industrial environments, a hybrid denoising approach combining wavelet threshold shrinkage and generative adversarial networks (WTS-GAN) is developed to optimize image quality before feature extraction. The encoded and enhanced images are then fused with raw time-series data through a lightweight dual-channel neural network, employing gated attention mechanisms to dynamically balance contributions from both modalities.

Discretization of Dirac Structures and Lagrange–Dirac Systems with Associated Variational Structures

Hiroaki Yoshimura

Waseda University

In this talk, we present the discrete geometry underlying Dirac structures and the associated Lagrange–Dirac systems, and we propose a discretization method for Dirac structures on manifolds. We then illustrate how nonholonomic systems can be formulated within the framework of discrete Dirac structures.

To this end, we begin by discretizing the canonical one- and two-forms on the cotangent bundle using finite difference maps of the (+), (−) type, which also serve to discretize the nonholonomic constraints. This allows us to define a discrete Dirac structure on the cotangent bundle.
Furthermore, we discretize the higher-order geometric structure known as Tulczyjew's triple on the cotangent bundle, and show that discretizing the Dirac differential of the Lagrangian yields a discrete Lagrange–Dirac system in the (+), (−) form.
Finally, we demonstrate the existence of a discrete Lagrange–d'Alembert–Pontryagin principle in the (+), (−) form, and show that the corresponding discrete equations preserve the discrete Dirac structure together with some examples of nonholonomic systems.
This is a joint work with Linyu Peng.

Generative Gaussian Splatting Flow

Zhengrui Xiang

Imperial College London

We investigate Generative Gaussian Splatting Flow (GGSF), a novel flow-matching approach for explicit 3D Gaussian splatting representations. Our method leverages Riemannian geometry to handle non-Euclidean features such as covariance matrices and rotation matrices, ensuring consistency with each primitive's underlying structure. We present preliminary experimental setups and outline directions for conditional extensions. We also investigate several advantages of the explicit flow, including editing and adaptive point cloud sampling.

16 July (Wednesday)

The de Casteljau algorithm for constructing cubic curves on Riemannian manifolds

Erchuan Zhang

Sun Yat-sen University

The classical de Casteljau construction of cubic curves has been generalised to a Riemannian setting since at 1980's, but the geometric properties of the resulting curves are still mysterious. In this talk, we will present the geometry buried in the de Casteljau construction of cubic curves on Riemannian manifolds. This talk is based on joint work with Prof. Lyle Noakes (The University of Western Australia).

Eigenvalue Problem of the p-Monge-Ampère Operator on Riemannian Manifolds

Tao Zheng

Beijing Institute of Technology

We consider the eigenvalue problem of p-Monge-Ampère operator especially for each $2 \leq p \leq n-1$. A notable characteristic of this equation is its degeneracy near the boundary, as the right-hand side tends to zero. Our approach is distinguished by the formulation of an algebraic inequality that incorporates the coefficients of the linearized operator. We establish the existence of the eigenvalue λ_1 and its associated eigenfunction u_1 within a smooth strictly p-convex domain Ω on a complete Riemannian manifold (X, g). This is a joint work of Jiaogen Zhang.

Application of Copula statistical model in machine vision

Chaorong Li

Yibin University

With the rapid development of machine vision technology, more and more signal processing technologies are being introduced into this field to cope with increasingly complex image recognition and analysis tasks. Copula, as a tool that connects the joint distribution of multiple random variables with their respective marginal distributions, can capture the nonlinear dependency structure between variables. Wavelet analysis, with its multi-scale time-frequency analysis capabilities, demonstrates a powerful local feature extraction capability in image processing. When Copula theory is combined with wavelet analysis, the two complement each other and can more accurately model and analyze complex structures and patterns in image data. This report focuses on the combination of Copula theory and wavelet analysis and its application in image classification and face recognition, as well as the design and use of Copula models within a deep network framework; this report also looks forward to the potential application of new technologies such as Copula Riemann manifolds and Copula multidimensional diffusion models in the field of machine vision.

Manipulating color representations within deep neural networks by computational persistent cohomology

Jiongyi Wang
Keio University

This presentation will explore the fascinating intersection of artificial intelligence and topological data analysis in understanding and manipulating color representations within deep neural networks, specifically focusing on CLIP's image encoder. We demonstrate that CLIP organizes colors onto a low-dimensional manifold with a discernible geometric structure, akin to the HSL color space.
A key aspect of our work involves the application of computational persistent cohomology to rigorously characterize the topology of this color manifold. This analysis quantitatively confirms a cylindrical structure, revealing how hues are distributed and how luminance varies. This topological understanding forms the foundation for the AI-driven manipulation of these color representations.
Building on this geometric insight, we show that color transformations can be achieved through simple linear operations within the embedding space of the AI model. By applying targeted transformations, for instance, using a single 3×3 matrix, we can perform effective color style transfer while crucially preserving the underlying image structure. This AI-driven approach has significant practical implications, particularly in biomedical imaging. We showcase its ability to address color inconsistency in cellular image segmentation, dramatically improving segmentation accuracy (e.g., from 0% to 80%) by aligning test image color distributions without requiring model retraining. These findings not only advance our comprehension of how AI models represent color but also offer an elegant and computationally efficient solution for color harmonization where structural integrity is paramount.

The Tropical Abel--Jacobi Transform of Metric Graphs

Yueqi Cao
Imperial College London

Metric graphs are widely used to model complex real-world data. The problem of how to extract and represent the geometric and topological information from graph data has given rise to various research areas, such as graph representation learning and graph reconstruction. Despite the immense literature in machine learning, the representation of metric graphs as abstract tropical curves has never been studied previously in computational and machine learning contexts. In this talk, I will introduce the tropical Abel--Jacobi transform of metric graphs and present algorithms for computation. Then I will discuss potential applications of the tropical Abel--Jacobi transform to graph embedding and topological data analysis.

17 July (Thursday)

Information-Theoretic Limits of Integrated Sensing and Communications Under Gaussian Channels

Fan Liu

Southeast University

Integrated Sensing and Communication (ISAC) is recognized as a promising technology for both the next-generation wireless networks and radar systems. In this talk, we consider a P2P ISAC model under vector Gaussian channels, and propose to use the CRB-rate region as a basic tool for depicting the fundamental sensing and communications (S&C) tradeoff. We characterize the S&C performance at the two corner points of the CRB-rate region. In particular, we derive the high-SNR communication capacity at the sensing-optimal point, and provide lower and upper bounds for the sensing CRB at the communication-optimal point. Our main results reveal a two-fold tradeoff in ISAC systems, consisting of the subspace tradeoff (ST) and deterministic-random tradeoff (DRT) that depend on the resource allocation and data modulation schemes employed for S&C, respectively. Finally, we investigate the generalized DRT based on the sensing mutual information from the perspective of rate-distortion theory.

Manifold Learning Based on Locally Linear Embedding for Symmetric Positive Definite Matrix

Hao Xu

China West Normal University

Symmetric positive definite (SPD) matrices have been widely utilized as feature descriptors in machine learning. To alleviate the computational burden associated with the high dimensionality of the original SPD matrices, dimensionality reduction (DR) techniques have been employed for SPD matrices extensively. However, traditional manifold learning algorithms for DR typically rely on the Euclidean metric and are primarily designed for vector-based data rather than SPD matrices. To address this issue, we propose a novel DR algorithm for SPD matrices named Lie-LLE, utilizing the basic idea of the locally linear embedding (LLE) manifold learning algorithm and leveraging the favorable properties of the Lie group structure inherent in the SPD matrix manifold. Specifically, the LLE algorithm is employed to perform DR process in the tangent space composed of symmetric matrices without transforming them into long vectors. Then, the corresponding SPD matrices can be obtained through the exponential map in the low-dimensional tangent space. Furthermore, Lie-LLE learns a mapping function that can directly process new samples, extending the theory and broadening the application of LLE.

Robust Weakly Coupled Markov Decision Processes

Fupeng Sun

Imperial College London

We study Robust Weakly Coupled Markov Decision Processes (RWCMDPs), a modeling paradigm for sequential decision-making under uncertainty in the transition kernel, where the global problem decomposes into independent subproblems coupled through linking constraints. The objective is to maximize the expected reward under the worst-case realization of the uncertainty for the finite horizon. We develop a general solution framework based on robust Lagrangian relaxation with state-action ambiguity sets. We prove that the resulting robust Lagrangian dual problem is non-convex and admits three equivalent formulations. These formulations enable the derivation of upper and lower bounds via subgradient methods and constant decision rule approximations. In a data-driven setting---where the ambiguity set is centered at the maximum likelihood estimator and converges in infinity-norm to the true transition kernel---we prove that the resulting policies constructed from both the upper and lower bounds are asymptotically optimal for the weakly coupled MDP under the true model, as both the ambiguity set radius and the number of subproblems grow large. Numerical experiments demonstrate the superior performance of our proposed methods.

Reconstruction of Graph Signals on Complex Manifolds with Kernel Methods

Yu Zhang

Beijing Institute of Technology

Graph signals are widely used to describe vertex attributes or features in graph-structured data, with applications spanning the internet, social media, transportation, sensor networks, and biomedicine. Graph signal processing (GSP) has emerged to facilitate the analysis, processing, and sampling of such signals. While kernel methods have been extensively studied for estimating graph signals from samples provided on a subset of vertices, their application to complex-valued graph signals remains largely unexplored. This paper introduces a novel framework for reconstructing graph signals using kernel methods on complex manifolds. By embedding graph vertices into a higher-dimensional complex ambient space that approximates a lower-dimensional manifold, the framework extends the reproducing kernel Hilbert space to complex manifolds. It leverages Hermitian metrics and geometric measures to characterize kernels and graph signals. Additionally, several traditional kernels and graph topology-driven kernels are proposed for reconstructing complex graph signals. Finally, experimental results on synthetic and real-world datasets demonstrate the effectiveness of this framework in accurately reconstructing complex graph signals, outperforming conventional kernel-based approaches. This work lays a foundational basis for integrating complex geometry and kernel methods in GSP.

Volume Discrepancy Measure on HPD Manifolds for Robust Target Detection

Jian-Yi Chen
Beijing Institute of Technology & Keio University

A novel robust object detection framework is proposed based on the volume discrepancy measure (VDM) on Hermitian positive-definite (HPD) manifolds. To address the performance degradation of traditional sample covariance matrix methods in non-homogeneous clutter environments and under limited sample conditions, a geometric detection method is established on HPD manifolds. Two VDMs are proposed to quantify the discrepancy between HPD covariance matrices. Based on determinant-based volume analysis, a VDM mean estimator preserving manifold geometry is developed and optimized via a trust-region algorithm incorporating the Riemannian gradient and Hessian. Simulation results in compound-Gaussian clutter environments demonstrate the effectiveness of the proposed framework. Furthermore, by introducing an orthogonal basis for Hermitian matrices, the associated influence functions are derived to further verify the robustness of the proposed approach under sample contamination. This talk is based on joint work with Bing-Zhao Li and Linyu Peng.

18 July (Friday)

Analysing Stability in Makespan Scheduling Algorithms

Xinyi Ye

Imperial College London

In real-world scheduling applications such as logistics and delivery, small delays in job processing times are common, yet fully recomputing optimal schedules in response is often impractical. While disruption management has been widely studied, most existing work focuses on preserving optimality, leaving a gap in understanding the stability of approximate methods commonly used in practice. This work studies the stability of some heuristics for the classical parallel-machine scheduling problem ($P \,||\, C_{max}$) when job processing times are changed. We present an approach to measure how stable scheduling algorithms are under such changes, using ideas from counterfactual analysis. We derive the conditions for perturbations to be permissible and define the concept of stability radius, as well as give exact algorithms to compute it for the Longest Processing Time (LPT) and the Local Search (LS) algorithms. We also study how both algorithms behave when all jobs are delayed equally (uniform perturbations), and give clear conditions under which their output does not change at all.

Beyond Adjacency: Graph Encoding with Reachability and Shortest Paths

Shiqiang Zhang

Imperial College London

Graph-structured data are central to many scientific and industrial domains, where the goal is often to optimize objectives defined over graph structures with specific node and edge configurations. Given the combinatorial complexity of graph spaces, such optimization problems are typically addressed using heuristic methods, and it remains unclear how to systematically incorporate structural constraints to effectively reduce the search space. In this talk, we introduce explicit optimization formulations for graph search space that encode properties such as reachability and shortest paths. We provide theoretical guarantees demonstrating the correctness and completeness of our graph encoding. To address the symmetry issues arising from graph isomorphism, we propose symmetry-breaking techniques to eliminate symmetries. Based on our graph encoding, we further formulate shortest-path graph kernels into a mixed-integer programming and develop a general graph Bayesian optimization framework for optimizing expensive black-box graph functions. Our framework enables global optimization over acquisition functions and is compatible with application-specific constraints. We demonstrate the effectiveness and versatility of our approach through two case studies: molecular design and neural architecture search, highlighting its potential in real-world applications.